# PERIODIC FADING IN OBLIQUE INCIDENCE SHORT-WAVE TRANSMISSIONS

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ABSTRACT. This paper deals with the interpretation of periodic fading observed in oblique incidence *CW* transmissions as due to changes in phase-paths of the interfering waves produced by critical frequency changes in the reflecting layer. Using Booker's equation as modified by Rao and Rao (1958), changes in path lengths and hence the number of interference maxima have been calculated and compared with the experimental results and a good agreement is observed. By comparing the theoretically calculated values with experimentally obtained ones due to the interference of the different modes, the possible modes by which the transmissions from Calcutta and Madras arrive at the receiving station (Waltair), have been determined. By using the number of interference maxima and the critical frequency values at any instant, a method of calculating critical frequency changes in small time intervals has been proposed.

## INTRODUCTION

Continuous wave-signal strength records due to radio-wave transmissions from distant stations usually show both random and periodic fading depending upon the ionospheric conditions. Appleton and Beynon (1947) observed and interpreted periodic fading of magneto-ionic origin. Periodic fading of a different origin was observed in oblique incidence short-wave *CW* transmissions by Banerjee and Mukherjee (1946) who interpreted this type of fading as due to continuously varying path difference between two interfering waves singly reflected and doubly reflected from a single layer having vertical movement or singly reflected from two different layers when one or both the layers undergo vertical movement. Later, Khastgir and Das (1950a) studied similar type of periodic fading in short-wave transmissions from distant stations and these authors have interpreted the observed fading as due to interference of two or more waves undergoing different Doppler frequency shifts when reflected from one or two ionospheric layers moving vertically. In a later communication, Khastgir and Das (1950b) have shown that these two apparently different interpretations are equivalent to each other.

In the present investigation the authors have attempted a quantitative interpretation of periodic fading observed in short-wave transmissions from distant A.I.R. stations, on the assumption that there is a continuous phase path change in one or both the ionospherically reflected waves due to ionisation changes.

## SOME RELEVANT THEORETICAL CONSIDERATIONS

For the purpose of interpreting quantitatively the observed periodic fading, the phase paths of the ordinary waves are calculated by using the approximate formula for phase-path developed by Rao and Rao (1958). The phase-path of an e.m. wave in the ionised medium is given by

$$\int \mu \, . \, ds \qquad \ldots \quad (1)$$

where $ds$ is an infinitesimal element along the path of the wave and $\mu$ is the phase refrective index at that particular element. According to the treatment of Booker (1939), $\mu'$ can be resolved into the vertical component $\mu_\psi \cos \psi (= q)$ and the horizontal component $\mu_\psi \sin \psi (= \sin i = s)$, where $\psi$ is the angle of refraction at the particular point under consideration and $\mu_\psi$ is the refractive index at the same point. Similarly, $ds = dh \cos \psi$, where $dh$ is an infinite simal element of height '$h$' at the same point.

Thus integral (1) can be expressed in terms of $q$ and $h$ as

$$\int \frac{q^2+s^2}{q} \, . \, dh \qquad \ldots \quad (2)$$

For the evaluation of the above integral, a knowledge of the variation of $q$ with $h$ is required. Booker (1939, 1949) had deduced for the case of obliquely incident radio waves a quartic equation giving the variation of '$q$', with $x$ for any given values of wave frequency '$f$', the earth's magnetic field $H$, the angle of incidence $i$, and the collisional frequency $\nu$. Rao *et al.* (1958) observed that the following empirical relation

$$q^2 = C^2 - \Delta x \qquad \ldots \quad (3)$$

gives close agreement with the $q$–$x$ curves as obtained by the quartic equation of Booker. The value of $\Delta$ is determined from the limiting values of $q$ and $x$ and is given by $C^2/x^1_r$, where $C$ is $\cos i$ and $x_r$ is the value of $x$ at the point of reflection i.e., at $q = 0$. If parabolic distribution of ionisation with height is assumed, then $x$ and $h$ are related by the well known expression

$$\beta h^2 - \alpha h + x = 0 \qquad \ldots \quad (4)$$

where $$\alpha = \frac{2f^2_0}{f^2 h_m}, \text{ and } \beta = \frac{f_o^2}{f^2 h^2_m}$$

where $f_0$ is the critical frequency of the ordinary ray, $f$ = operating frequency, $h_m$ is the semi-thickness of the layer. Using the above relations (3) and (4), $q$ can be expressed in terms of '$h$' alone and the integral (2) can now be written as

$$\int \frac{C'^2\Delta(\beta h^2 - \alpha h) + S'^2}{[C'^2 + \Delta(\beta h^2 - \alpha h)]^{\frac{1}{2}}} \cdot dh \qquad \ldots \quad (5)$$

As the above expression involves only one variable $h$, it can be evaluated between the required limits. Actual integration and simplification gives the final equation for the phase path $P$ as

$$P = C'h_m + h_m \cdot \frac{f}{f_0}\left[2 - \left(C'^2 - \frac{f_0^2}{f^2}\right)\right] \ln \frac{1+D}{1-D} \qquad \ldots \quad (6)$$

where $$D = \frac{f \cdot \cos.\, i}{f_0}$$

Using the above expression, phase paths for the ordinary ray are calculated at two different times knowing the values of critical frequencies at those times. The change in phase path of any interfering mode due to the varying electronic density is thus obtained. By estimating the change in the phase path difference between the two interfering modes, the number of fading maxima expected to occur in that interval of time can be calculated.

## EXPERIMENTAL DETAILS AND RESULTS

Using an Eddystone communication receiver of the model $S$-504, with a D.C. amplifier and an Esterline-Angus pen recorder, periodic fading in short wave transmissions of 6.085Mc/sec from Madras, and 7.21 Mc/sec from Calcutta at distances of 620 and 730Km respectively from the receiving station (Waltair) has been studied.

The records have been taken during the early afternoon hours from 1230 to 1500 hrs I.S.T. with a view to minimise the contribution to path changes due to the vertical movement of the reflecting layers. The heights and semi-thicknesses assumed for the $E$, $F_1$ and $F_2$ layers are 100Km and 20Km, 220Km and 60Km, and 320Km and 140Km respectively. The vertical equivalent frequencies (f. cos. $i$.) for the different layers for the different modes of propagation for both the stations, Madras and Calcutta, are presented in Table I.

Preliminary experimental investigation has been made with transmissions on 6.085Mc/sec from Madras between 1300 and 1400 hrs. I.S.T. The critical frequencies for the $E$ layer at these times during which records have been taken are 3.4Mc/sec and 3.3Mc/sec and for $F_1$ layer during those hours are 4.6 and 4.5 Mc/sec respectively. Typical results of calculations made for a record taken on 14.2.'55 are given below. From considerations of the $f$. cos. $i$. values given in

TABLE I

| Station | Operating frequency f | Distance from the receiving station | f. cos. i. values | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | | 1E | 2E | $1F_1$ | $2F_1$ | $1F_2$ | $2F_2$ |
| Madras | 6.085 Mc/s | 620 Km | 1.86 | 3.30 | 3.52 | 4.98 | 4.37 | 5.51 |
| Calcutta | 7.21 Mc/s | 730 Km | 1.90 | 3 47 | 3.72 | 5.56 | 4.76 | 6.26 |

Table I, $1E$, $2E$, $1F_1$ and $2F_2$ modes of propagation are possible. The interference between any two of the above possible four modes gives rise to periodic fading. Actual calculation and a comparison of those results showed that interference between $1E$ and $2E$, and $1E$ and $1F_1$ are only present. The frequency of fading for $1E$ and $2E$ interference observed is 16 as against the calculated value of 22. The observed value of frequency of fading for $1E$ and $1F_1$ interference is 2 as against the calculated value of 1.44. The critical frequency data are taken as reported from the Ahmedabad Ionospheric Station because no critical frequency data are available from Madras for the $E$ and $F_1$ layers. The agreement between the calculated and the observed values of fading frequency may be considered as good in view of the approximate values assumed for the critical frequencies.

Further investigation on these lines has been carried out extensively using transmissions on 7.21 Mc/s from Calcutta station between 1230 and 1330 hrs. I.S.T., in the months of November and December, 1956, and between 1400 and 1500 hrs. I.S.T. during the months of February and March, 1957. The heights and semi-thicknesses of the different layers are assumed as before and the $f$. cos. $i$. values for single and double hop. reflections for the different layers are as presented in Table I. Comparing the vertical equivalent frequencies for $1E$, $1F_1$ and $1F_2$ modes of propagation with the $f_0E$, $f_0F_1$ and $f_0F_2$ values at those hours, it has been found that $1F_1$ and $1F_2$ modes are not possible, as each mode suffers reflection from the lower layer. Further the $2F_1$ mode is not possible as the equivalent frequency for this mode is found to be very close to and sometimes less than $f_0F_1$. The $2F_2$ mode, though theoretically possible, is likely to suffer very large deviative and non-deviative absorption and hence it is unlikely to be present in significant strength. Thus the single and double hop reflections from $E$ layer will be the predominant modes of propagation for transmissions from Calcutta received at Waltair. The experimental fact that most of the records show simple fading patterns with a single periodicity of large amplitude confirms the assumption that $2F_2$ mode is not present in significant strength. Two such typical records of this type are shown in Eig. 2. Complicated patterns indicating superposition of more than one periodicity appeared only occasionally. Phase paths and frequencies of fading have been calculated for the months of November and December using relation (6) and the critical frequency data are taken from the

Ionospheric Research Station, Ahmedabad. The $f_0E$ values at Ahmedabad are found to be lower than those at Waltair and higher than those at Calcutta by about 0.1 Mc/sec. Hence the Ahmedabad data are taken to represent fairly well the conditions existing at the reflecting point as its latitude is midway between those of Calcutta and Waltair. Taking any two of the three possible

TABLE II

The results of theoretical calculations of interference maxima in fading records and comparison with experimental values—Calcutta -7210Kc/sec.

| Date | Beginning time of the record hrs. | Time duration mts. | Critical frequency E layer | | No. of interference maxima per minute | |
|---|---|---|---|---|---|---|
| | | | At 1230 hrs. in Mc/sec. | At 1330 hrs in Mc/sec, | Calculated | Observed |
| 20.11.'56 | 1258 | 5 | 4.00 | 3.85 | 9.0 | 11.4 |
| 20.11.'56 | 1312 | 5 | 4.00 | 3 85 | 9.0 | 13.8 |
| 21.11.'56 | 1234 | 5 | 3 95 | 3.80 | 10 0 | 12.6 |
| 22.11.'56 | 1243 | 5 | 4.00 | 3 90 | 6 0 | 8.4 |
| 23.11.'56 | 1237 | 6 | 3.95 | 3.80 | 10.0 | 13.8 |
| 26.11.'56 | 1251 | 5 | 3.95 | 3.75 | 14.2 | 13.3 |
| 27.11.'56 | 1238 | 10 | 3 95 | 3.80 | 9 0 | *17 2 |
| 30.11.'56 | 1238 | 4 | 4.10 | 3 90 | 10.0 | *17.5 |
| 30.11.'56 | 1243 | 5 | 4.10 | 3 90 | 10.0 | *16.3 |
| 3.12.'56 | 1252 | 11 | 3 95 | 3 80 | 10.0 | 11 8 |
| 4.12.'56 | 1243 | 10 | 3.95 | 3.80 | 10 0 | 13.5 |
| 11.12.'56 | 1245 | 6 | 3.95 | 3.80 | 10.0 | 11.8 |
| 12.12.'56 | 1243 | 5 | 4.00 | 3.85 | 9.0 | 8.6 |
| 20.12.'56 | 1251 | 10 | 3 95 | 3 80 | 10.0 | *16 0 |

*Presence of $E_s$ suspected.

modes of propagation, the difference in the phase paths and hence the frequency of fading per minutes have been calculated. The calculated frequency of fading is about 1C/mnt. for $1E$ and $2F_2$ interference, about 12C/mnt. for $1E$ and $2E$ interference and about 20C/mnt. for $2E$ and $2F_2$ inteference. There is fairly wide variation in these values depending upon the critical frequency values of $E$ and $F_2$ layer on the particular day of observation. The observed frequency of fading centres round the value 13.00C/mnt. for most of the days. Thus it is evident that $1E$ and $2E$ interference is mainly responsible for the observed periodic fading. The results of the detailed calcuiations for the indivi-

TABLE III

Results of critical frequency changes in *E* layer deduced from fading records and comparison with those deduced from critical frequency data

| Date | Starting time of record hrs. I.S.T. | Observed no. of peaks per minute | Time duration in minutes | $f_0E$ in Mc/s 1400 hrs | $f_0E$ in Mc/s 1500 hrs | Reference frequency in Mc/s | Change in $f_oE$ in Kc/s during the time of record. Deduced from fading records | Change in $f_oE$ in Kc/s during the time of record. From $f_0E$ data |
|---|---|---|---|---|---|---|---|---|
| 7.2.'57 | 1426 | 19.0 | 6 | 3.8 | 3.6 | 3.70 | 21 | 20 |
| 12.2.'57 | 1421 | 24.8 | 4 | -- | -- | --- | — | — |
| 16.2.'57 | 1416 | 12.3 | 12 | 3.9 | 3.7 | 3.8 | 36 | 40 |
| 20.2.'57 | 1426 | 14.4 | 7 | 3.9 | 3.6 | 3.8 | 24.5 | 35 |
| 25.2.'57 | 1443 | 15.4 | 8 | 3.8 | 3.6 | 3.75 | 24 | 26.7 |
| 26.2.'57 | 1424 | 11.1 | 5 | 3.8 | 3.6 | 3.75 | 10 | 16.7 |
| Do | 1452 | 14.6 | 8 | 3.8 | 3.6 | 3.70 | 24 | 26.7 |
| 27.2.'57 | 1410 | 13.0 | 8 | 3.9 | 3.6 | 3.85 | 32 | 40 |
| 1.3.'57 | 1419 | 10.0 | 6 | 3.8 | 3.6 | 3.80 | 15 | 20 |
| Do | 1453 | 10.0 | 7 | 3.8 | 3.6 | 3.65 | 17.5 | 23.3 |
| 3.3.'57 | 1404 | 12.4 | 5 | 3.9 | 3.8 | 3.90 | 15 | 8.3 |
| Do | 1412 | 9.1 | 10 | 3.9 | 3.8 | 3.80 | 20 | 16.7 |
| 4.3.'57 | 1430 | 17.5 | 8 | 3.8 | 3.6 | 3.70 | 28 | 26.7 |
| 5.3.'57 | 1414 | 14.4 | 8 | 3.9 | 3.7 | 3.75 | 24 | 26.7 |
| 6.3.'57 | 1450 | 11.5 | 10 | 3.9 | 3.8 | 3.80 | 25 | 16.7 |
| 7.3.'57 | 1445 | 12.3 | 8 | 4.0 | 3.8 | 3.90 | 28 | 26.7 |
| 8.3.'57 | 1411 | 15.3 | 6 | 4.0 | 3.8 | 3.95 | 30 | 20 |
| Do | 1438 | 13.3 | 10 | 4.0 | 3.8 | 3.90 | 40 | 33.7 |
| 9.3.'57 | 1415 | 21.6 | 5 | 3.7 | 3.6 | 3.80 | 25 | 25 |

dual days are shown in Table II. An examination of Table II shows that the observed frequency of fading is agreeing fairly well with the theoretically calculated value for $1E$ and $2E$ interference for most of the days. In view of the approximation involved in the phase-path formula and lack of critical frequency data from a station close to the point of reflection, the agreement may be considered as reasonably good. However, weak signals by $2F_2$ mode are received occasionally as evidenced by a secondary periodicity superposed on the common type periodic fading.

## A METHOD OF DETERMINING CRITICAL FREQUENCY CHANGE FROM THE OBSERVED PERIODIC FADING PATTERNS

The above study has suggested to the authors the possibility of making use of such experimentally obtained periodic fading records for the study of minute ionisation changes in the reflecting layer. The rate of change of path difference with time between the two modes suffering reflection from the same layer and interfering to produce fading is dependent upon the ionisation changes in the reflecting layer, provided the layer height remains the same. Thus the observed fading period is related to the critical frequency change.

The path difference between $1E$ and $2E$ reflections expressed in terms of operating wavelength is calculated for different values of $f_0$ of the reflecting layer and a graph is drawn between this path difference and the corresponding

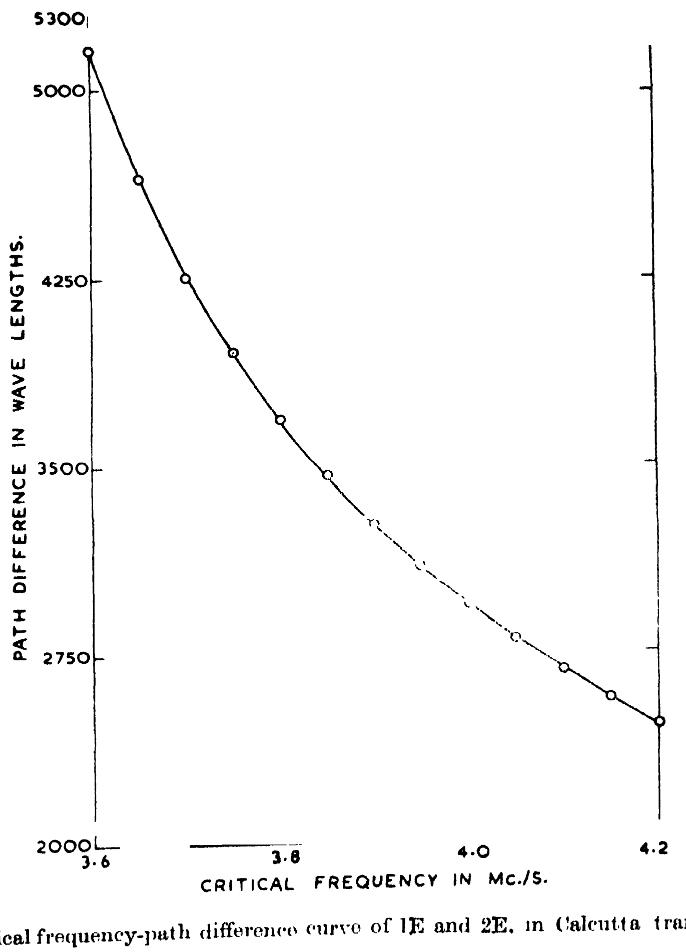


Fig. 1. Critical frequency-path difference curve of 1E and 2E. in Calcutta transmissions on 7210Kc/sec.

$f_0$ values as shown in Fig. 1. If a fading record is taken in a known interval of time and if the critical frequency of the $E$ layer is known either at the beginning or end of the record, the critical frequency corresponding to each fading at each interval of one cycle can be calculated as well as the critical frequency change for the duration of the record.

This latter method has been used by the authors to evaluate critical frequency changes in short intervals of time from the observed fading periods. The critical frequency change during the short interval of the record is read from Fig. 1, assuming the $f_0$ value at the beginning of the record. This value is then compared with the expected $f_0E$ change in that interval, calculated from hourly values of

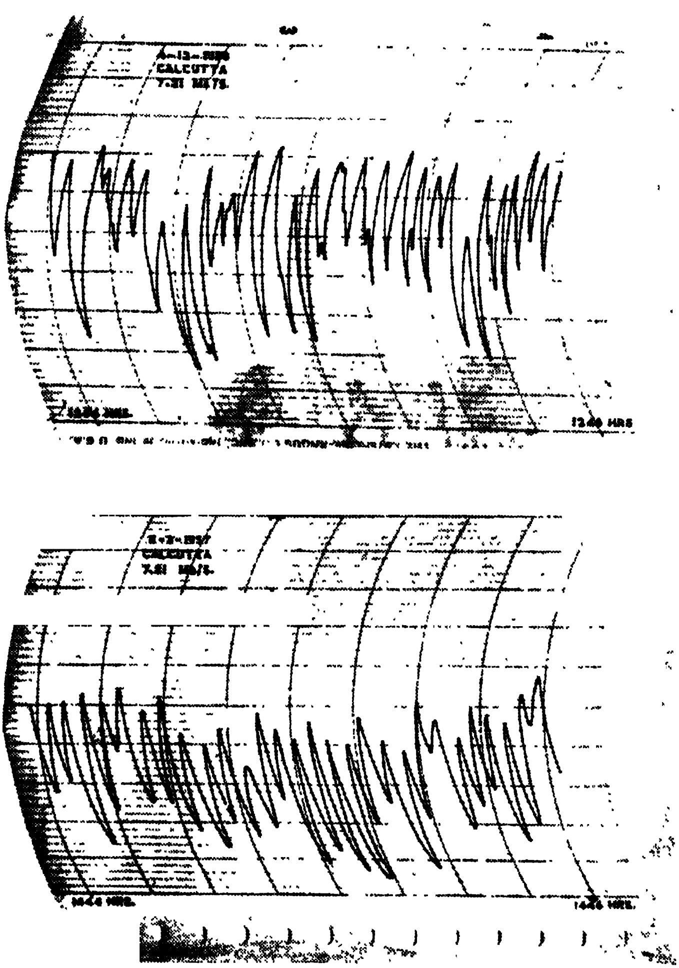

Fig. 2. Periodic fading in Calcutta transmissions on 7210Kc/sec. received at Waltair, due to the interference between 1E and 2E reflections.

$f_0E$ given in Ionospheric Data Bulletin assuming linear variation of $f_0E$ with time in one hour interval.

A large number of records were taken during February and March, 1957 for Calcutta transmission on 7.21Mc/sec between 1400 and 1500 hrs. I.S.T. and as in the previous months, it is found that $1E$ and $2E$ interference is the predominant cause of periodic fading. The contribution to phase path change due to height variation is found to be negligible compared to that due to critical frequency change. The results of the calculation are presented in Table III.

An examination of the above Table III shows that there is good agreement between the theoretically calculated and experimentally observed values. This confirms the correctness of the interpretation of the periodic fading as due to the interference of $1E$ and $2E$ reflections. As the change in critical frequency deduced from the graph depends not only upon the frequency of fading, but also upon this reference critical frequency, any inaccuracy in choosing this reference frequency will introduce some error in the calculation of the critical frequency change. One significant fact to be noted is that the day to day fluctuations in the critical frequency values and the magnitudes of their change are regularly observed as a change in the frequency of fading.

The present method of interpretation adopted by the authors has the advantage that the observed fading period is directly related to the critical frequency change of the ionospheric layers and hence it has been possible to estimate particularly the short period critical frequency changes from the fading records. The method has the advantage that the technique is simple.

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